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"Accumulation of Return And Risk Of An Investment Portfolio"

Technical Field

The invention concerns a method and computer system of determining the
5   accumulation of expected return and associated risk of an investment portfolio.

Background Art

Modern portfolio theory (MPT) is concerned with the trade-off between
investment return and risk, mostly with a single-period focus. One outcome has been a
10   process that involves selecting assets in certain proportions that complement each other
in order to mitigate risk while attaining risk objectives, or maximising returns for a
given risk tolerance. This process is known as portfolio asset allocation.

Numerous models have been developed to assess relationship between expected return
and risk for investment portfolios. The most famous model is the mean-variance
15   optimisation (MVO) developed by Markowitz, which describes the change in risk and
return as the asset allocation of the portfolio is varied. This model is concerned with
risk and return for a single, predetermined time period. When applied to superannuation
portfolios MVO typically centres on annual return and annual risk. This limited
treatment of risk is inconsistent with superannuation investing of the defined
20   contribution type which seeks to build wealth over long and variable investment
horizons. That is, current practice focuses on risk and return at the early stages of the
investment whereas it is in the interests of the investor to understand expected wealth
and risk outcomes at the end of the maturing of the investment and along the
intervening pathway. No method exists to satisfactorily assess risk as an investment
25   accumulates return from a portfolio of assets and periodic capital contributions over
time. This is despite the burgeoning of defined contribution superannuation investment
with the purpose of wealth accretion over investment periods of many decades.

Typically when accumulation of wealth from investment return, ret, is addressed
over varying time periods, n, the common practice is simply to compound expected
30   returns:

$$\text{Wealth} = \text{cap}*(1+\text{ret})^n$$

where cap is the initial value of a portfolio.

The issue is generally treated deterministically. That is, no attempt is made to
assess how the risk to wealth changes over time, even though returns are probabilistic
35   in nature. Introducing risk requires that asset-specific characteristics be identified and
modelled over time.

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After Markowitz, MPT has assumed that risk of an asset's returns is given by the probability distribution of returns, and that this can be described by the normal or Gaussian distribution (symmetrical bell shape). It has also been assumed that returns follow a random walk (i.e. that there is no correlation between returns in subsequent periods). If that is indeed the case then modelling asset risk and return behaviour over time would not be difficult. The risk at any year, t, beyond one year R(t) can be calculated analytically from the annual risk, R(1) as:

$$R(t) = R(1)/sqrt\ (t)$$

However both of these assumptions (normality and random walk) have been shown to be questionable over longer time frames. The literature on this aspect has developed within the last decade. The distributions observed can be skewed and the serial correlation between asset returns quite marked. This serial correlation results in phenomena such as the mean-reversion observed in the returns of most share markets over mid to long term investment horizons. The importance of these effects in superannuation is reinforced by its very nature of regular and variable capital contributions over time and 'locking-up' of these investments until retirement.

Superannuation funds, sometimes known as wholesale funds, that comprise a plurality of investments on behalf of a plurality of investors focus on the performance of the fund at a fund level. The fund's board will set investment objectives for the fund, such as maximising return subject to some broad risk constraint. Using various tools such as MVO and individual judgement a strategic asset allocation (SAA) will be derived for the entire fund.

For example, a SAA could be derived that requires 70% of the capital to be invested in shares and 30% of the capital invested in bonds. According to this SAA, the fund's capital at that time (t=1) will be allocated to these investment categories, in those proportions.

At t=1 (the first year), the capital of the fund would be owned by its investors in different proportions according to their contributions. For example, for a fund that has a capital of $1000, investors A, B and C would have the following different financial weights based on the following contributions:

Investor A has $200 - weight in fund = 20%

Investor B has $300 - weight in fund = 30%

Investor C has $500 - weight in fund = 50%

At t=2 (the second year) the board updates each investor's capital according to the return experienced by the fund on the whole and the individual weights of the

investors. For example, if the return on investment was 10% net the new capital amounts at t=2 for each investor would be:

Investor A = (0.2 * 1000) * (1 + 0.10) = $220

Investor B = (0.3 * 1000) * (1 + 0.10) = $330

Investor C = (0.5 * 1000) * (1+ 0.10) = $550

It can be seen that following this method, each investor has identical proportional increase in capital and no factor other than an investor's financial weight in the fund is taken into account when distributing the return.

A SAA for the fund is devised for that year (t=2) and the cycle continues.

Investment products, such as those offered by retail channels and also in wholesale superannuation provide 'Investment Choice' menus, are characterised by intuitive risk descriptions (conservative, aggressive, low or moderate chance of negative return etc). These descriptors usually are derived from measures of expected annual return and risk. Investors in such products are also rewarded simply on the basis of financial weight in the product pool.

Under the current practice substantial wealth is expected to be foregone by most investors, usually at greater risk.

Summary of the Invention

In a first aspect, the invention provides a method for determining expected accumulated return and associated risk of an investment having an investment term, the investment term comprised of a plurality of periods, the method comprising:

calculating an expected annualised asset return distribution for an asset over different holding periods of different length;

sampling the expected annualised asset return distribution for the holding period substantially equal to the investment term to extract a single expected return on initial capital of the investment;

for expected returns generated in each period, sampling the expected annualised asset return distribution for a holding period substantially equal to the total of the remaining periods of the investment term to extract a single expected return on each return previously generated;

summing each of the sampling extractions and storing the result representing a single expected return for the investment;

repeating each of the sampling and summing steps; and

determining the expected accumulated return and associated risk of the investment using the results.

It is an advantage of the invention that the behaviour of assets over long term investment horizons can be accounted for when determining the expected return and associated risk of an investment. Using stochastic simulation to sample the asset return distributions in this manner allows for asset specific correlations and effects such as

5   non-Gaussian distributions, serial correlation, mean reversion and aversion of returns. It is a further advantage, that the method identifies the different components of the investment, such as initial capital and expected returns according to 'time-of-entry', that is a period within the investment term.

The investment term may be equal to the investment life of the investment. It is

10   an advantage of at least one embodiment of the present invention that determinations of expected accumulated return and associated risk can be made for the investment life of the investment. As a result, advice can be given to investors that does not follow a limited focus that is applicable only to a minority of investors.

Each period may be equal in length. Each period may be a year in length. The

15   first holding period of the different holding periods may be equal to the investment term. Further different holding periods may be each holding period progressively smaller than the first holding period by a period. The calculated asset return distributions may be based on the observed past performance of the asset.

The determined expected accumulated return of the portfolio may be used to

20   calculate expected accumulated wealth distribution of the investment.

Sampling the expected annualised asset return distribution for a holding period may comprise sampling the expected annualised asset return distribution a number of times equal to the number of periods within that holding period. An extract of a single expected return may be a combination of the number of samples.

25   The investment may be a superannuation investment or an investment in a managed fund. The sampling and summing steps may be repeated over a 100 times, such as 5,000 times.

The method may further comprise also determining the expected accumulated return and associated risk of an investment over a smaller investment term also

30   comprised of periods. This is done by performing the sampling, summing, repeating and determining steps of the method using a smaller investment term as the investment term. The expected return and associated risk of an investment may be determined for each smaller investment term within the investment term, starting from a first smaller investment term substantially equal to a single period, then each term progressively

35   larger than the first smaller investment term by a period. This gives the investor a

better understanding of the progressive expected accumulated return and associated risk of an investment over any investment term.

The investment may include capital contributions made for any period within the investment term. The method may further comprise for each contribution made in a period, sampling the expected annualised asset return distribution for a holding period substantially equal to the total of the remaining periods of the investment term to extract a single expected return on each contribution.

The expected return generated in a period may be the total of the expected returns generated in that period from the initial capital and the expected returns generated in that same period from any returns previously generated. The expected returns generated in a period may also include the expected returns generated in that same period from contributions made.

The asset return distribution may be derived from standard market indices, or be a subset of standard indices selected on the basis of parameters such a price to book value, sectoral bias or other 'active' tilts. The asset return distribution for any marketed investment product may be derived from the published construction of the product.

The investment may be made in one or more assets. Proportions of the investment divided into the different assets may represent a strategic asset allocation of the investment. The step of calculating expected annualised asset return distribution for an asset may be performed for each asset that is included in the strategic asset allocation of the investment. The sampling steps may be repeated for each asset that is included in the strategic asset allocation of the investment, based on that asset's calculated expected annualised asset return distributions.

Where the investment has more than one asset, after extracting a single expected return on initial capital for each asset, the method may further comprise combining the expected returns for each asset according to each asset's weight allocation within the strategic asset allocation to calculate a single expected return on initial capital of the investment for that strategic asset allocation.

Where the investment has more than one asset, after extracting a single expected asset return on each previously generated return for each asset, the method may further comprise combining the expected return on each previously generated return of each asset according to each asset's weight allocation within the strategic asset allocation to calculate a single expected return on each return previously generated for that strategic asset allocation.

The method may be repeated based on different strategic asset allocations for the investment. This gives the investor advice on the expected return accumulation and

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wealth accumulation and associated risk for different investment strategies comprising different asset allocations, different initial capital amounts and different contributions over varying investment horizons. An actual strategic asset allocation for the investment may be chosen by comparing the expected accumulated return and wealth and associated risk of an investment as determined for each strategic asset allocation. A strategic asset allocation may be chosen based on the best performance, the risk tolerance of the investor or the return and wealth expectations of the investor.

The step of using the results to determine the expected accumulated wealth and associated risk of an investment may comprise graphically representing the results. For example, the results could be plotted on a graph having expected wealth and frequency as the axes. The expected accumulated wealth of an investment may be determined to be the mean or median of the results determined by the method. The associated risk of an investment may be determined based on the spread of expected wealth determined by the method.

The step of calculating a distribution of expected annualised asset returns for an asset over different holding periods of different length may comprise:

calculating an expected annualised return for an asset derived from an estimated risk premium for that asset;

calculating a representative annualised return distribution for an asset over the different holding periods;

for each representative annualised return distribution, calculating the likelihood of degrees of variation from a central tendency; and

combining the expected annualised return for the asset and the variations calculated for each holding periods into the distribution of expected annualised asset returns for holding period.

The expected returns may be real returns or may be nominal returns.

The method may further comprise collecting details on the investor. The strategic asset allocation for an investment may be determined based on the details of the investor. The investor details may be any one or more of the investor's age, expected membership duration (i.e. for a superannuation investment this would be expected contribution life or retirement age, for a managed fund that would be the expected investment term), income, current investment capital with the fund, contributions amount, prospective capital additions and withdrawals, wealth objectives, risk tolerance for expected wealth, other major investments, taxation and other special circumstances. It is an advantage of at least one embodiment of the invention that practically draws longer term asset effects into a wealth accumulation strategy,

optimising individual needs, that would enable the superannuation industry truly to discharge its obligation to defined contribution investors of maximising wealth accumulation.

The method may further comprise periodically recalculating the expected
5   annualised return distributions.

The method may also comprise automatically updating a strategic asset allocation of an investment, such as based on the inventor's details or new expected annualised asset return distribution. It is an advantage of at least one embodiment of the present invention that a strategic asset allocation is kept optimal by periodically
10 adjusting it, for example the acceptable risk for an investment may reduce as the investor gets older.

The method may further comprise determining a strategic asset allocation by selecting a first asset, then selecting one or more other assets that most exploits the relationship between return and risk of the investment to the investor's advantage.
15 Optionally, the combination of assets should not exceed the risk tolerance of the investor. The first asset may be a domestic asset that has the highest expected rate of return over the investment term. Alternatively, the first asset may be an asset that is substantially equal to the risk tolerance of the investor.

Optionally the method may further comprise determining a strategic asset
20 allocation for an investment fund having a plurality of investments, by:

determining a strategic asset allocation for each of the plurality of investments according to the method described above; and

determining a strategic asset allocation for the fund using an aggregate of the strategic asset allocation for each of the plurality of investments.

25   By determining the investment portfolio based on the characteristics of the investors of the fund rather than the desired overall performance of the fund, the investment portfolio of the whole fund is pitched at the individual needs of the members rather than a fund level which aims to have the overall fund perform well. This method is the reverse of current schemes that derive a strategic asset allocation for
30 the entire fund based on broad fund objectives and then distribute annual returns to the investors based on their proportion of the fund's capital.

The method may further comprise considering the fund's constraints, such as liquidity, asset limits, tax, when determining a strategic asset allocation for the fund.

The method may further comprise allowing the investor to amend their strategic
35 asset allocation. For example, after receiving the expected accumulated wealth and associated risk of a strategic asset allocation, the investor may request that the strategic

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asset allocation be amended to meet a different assessment of risk or different asset allocation.

A sequential, multi-period model for superannuation current practice of investment strategy exploits these asset-specific phenomena, in a measured way, in the

5   investors interest.

In a second aspect, the invention provides a computer system to determine expected accumulated return and associated risk of an investment over an investment term, the investment term comprised of a plurality of periods, the system comprising:

an asset datastore to store an expected annualised asset return distribution for an

10 asset over different holding periods of different length;

a processor to repeatedly sample the expected annualised asset return distribution for the holding period substantially equal to the investment term to extract a single expected return on initial capital of the investment; and for expected returns generated in each period, repeatedly sampling the expected annualised asset return

15 distribution for a holding period substantially equal to the total of the remaining periods of the investment term to extract a single expected return on each return previously generated;

a sample datastore to store a sum of each repeat of the sampling extractions which represents a single sample expected return for the investment;

20 wherein the processor uses the sample datastore to determine the expected return and associated risk of the investment.

Brief Description of Drawings

An example of the invention will now be described with reference to the

25 accompanying drawings, in which:

Fig. 1 is a graph showing the annual returns for Australian shares from 1925 to 2001;

Fig. 2 is a graph showing the annualised returns for Australian shares over thirty year holding periods;

30 Fig. 3 is a graph showing the annual returns for Australian ten year bonds from 1925 to 2001;

Fig. 4 is a graph showing the annual returns for Australian ten year bonds over thirty year holding periods;

Fig. 5 is a simplified flowchart of the method of the invention;

35 Fig. 6 is a simplified representation of the investment components over a four year investment horizon;

Fig. 7 is a graph showing the distribution of expected wealth at t=3 after 5000 simulation runs;

Fig. 8 is a graph showing the wealth creation pathway for Investor A;

Fig. 9 is a graph showing the wealth creation pathway for Investor C;

5   Fig. 10 is a graph showing the wealth distribution after one year using a Typical SAA;

Fig. 11 is a graph showing the wealth distribution after one year using a Option A SAA;

Fig. 12 is a graph showing the wealth distribution after five years using the 10   Typical SAA;

Fig. 13 is a graph showing the wealth distribution after five years using the Option A SAA;

Fig. 14 is a graph showing the wealth distribution after five years using the Typical SAA for an investor with modest initial capital;

15   Fig. 15 is a graph showing the wealth distribution after five years using the Option A SAA for an investor with modest initial capital; and

Fig. 16 is a flow diagram showing the method of determining the SAA of an investment fund having a plurality of investments.

20   **Best Mode for Carrying Out the Invention**

Using the invention, assets are defined principally by expected return and risk at different periods within an investment term, such as forecast investment life. Asset riskiness at each yearly interval is mainly defined by the observed historic distribution of annualised returns. Distributions of asset returns can be skewed and the serial 25   correlation between an asset's returns over time quite marked. The distributions of an asset may vary over different holding periods.

For example, Fig. 1 shows the distribution of annual returns for Australian shares from 1925 to 2001. This graph shows that representing this behaviour with a Gaussian distribution is reasonable. Fig. 2 shows the distribution of annualised returns 30   over thirty year holding periods for Australian shares. Comparing Figs. 1 and 2 shows that the risk to annualised return is different for Australian shares over the different holding periods as seen from the different dispersions.

The same analysis can be performed on the returns for Australian bonds. Fig. 3 shows the distribution of annual returns for Australian ten year bonds from 1925 to 35   2001. Fig. 3 appears skewed to the right making a Gaussian assumption questionable. Fig. 4 shows the return of Australian ten year bonds for holding periods of 30 years.

As the distributions of Figs. 3 and 4 are different, it shows that risk over different holding periods is also different for bonds. Some difference is to be expected from random walk theory as the holding period in increased but not to the extent observed here. Comparing Fig. 4 to Fig. 2 shows that shares have not been as risky for investors

5   as bonds when the investor is concerned with a 30 year holding period, as the dispersion in Fig. 2 is minor. This long term result is contrary to conventional MPT that assumes bonds are less risky, as a result of the yearly focus inherent in current finance industry practice serving superannuation investors. This difference is due to mean reversion in shares and mean aversion in bonds over longer investment horizons.

10   These effects are significant to investors. Modelling these effects on the scale larger than a year, such as over 30 years for about 10 assets would be impossible analytically. The technique of stochastic simulation enables these influences to be modelled across assets, time and the portfolio components having different time-streams in the wealth creation process. That is done by assigning a 'time of entry' for

15   funds invested along the investment term and assigning risk according to the time of entry in relation to the investment life. In a superannuation investment, the investment life usually would be equivalent to the investor's expected remaining working life. For example, initial starting capital is treated separately from contributions, and contributions in each year are treated separately and the returns to the portfolio in each

20   year are treated separately. Referring to Fig. 5, this process will now be explained in further detail.

Initially 60, a single annual expected return for each asset (a, b, ...) of interest to the investment is derived from an estimated risk premium for that asset. For example:

$$ret_a = risk\ premium_a + inflation\ expected + time\ value\ of\ money$$

25    $$ret_b = risk\ premium_b + inflation\ expected + time\ value\ of\ money$$

etc ...

The expected return on the portfolio (r_{port}) is then the weighted sum of the assets. It can be seen that these risk premiums and resulting r_{port} values are based on nominal returns. Alternatively, the method could be performed on real returns where inflation is

30   not taken into consideration.

Representative distributions are then calculated for annualised returns for each asset for holding periods of different length each ending at the end of the investment term. The difference between two successive holding periods is one period, of typically a year. For example, for an investment that has a life of 30 years, an expected

35   return distribution for a holding period of 30 years, 29 years and every successive year down to a one year holding period would be calculated for each asset of interest to the

investment.   The holding period of 29 years would be applied to investment components entering the investment stream at the beginning of the second year and end at the end of the 30th year. The one year holding period would start on the beginning of the 30th year of the investment and end at the end of the 30th year of the investment.

5   Details on the past performance of investible assets are used to calculate the asset distributions. Possible details include:

        past returns

        risks

        liquidity

10   correlation of returns between assets

        correlation of returns of an asset over different periods.

        This information can be collected using publicly available information and in-house research.

        Then for each representative distribution the likelihood of degrees of variation

15 from a central tendency measure for each distribution is calculated. New distributions for the expected annualised asset returns for each holding period are then calculated by placing the variations calculated for each holding period of that asset above around the calculated single expected return.

        Expected return distributions for an investment portfolio of assets are derived

20 from sampling randomly the relevant asset distributions. The procedure could be summarised as follows:

        The return on an investment portfolio in any period Ret(port) is calculated from:

        Ret(port) = SUM (ret(cap) + SUM ret(rets) + SUM ret(conts))

Where:

25   ret(cap) is the return on initial starting capital. E.g. at t=1 for a thirty year investment the return on initial capital for that year is obtained by sampling from the 30 year distributions of assets;

        ret(rets) is the return on the portfolio from each preceding year. E.g. the return on the return generated at t=1 is obtained by sampling from the 29 year distributions of

30 assets. The SUM denotes that to this is added the return on the portfolio at t=2 which is sampled from the 28 year distribution and so on;

        ret(conts) is the return to the contributions made in a period and would be treated in the same way as the returns in ret(rets) above. E.g. for a superannuation investment, this would be the investors and the employer's yearly contributions.

35   The following example explains in detail how the accumulated wealth of an investment portfolio and associated risk is estimated at yearly intervals for a three year

investment term and a portfolio of two assets a and b, with weights w_a and w_b rebalanced annually. Fig. 6 shows a simplified schematic representation of the investment components over an investment horizon.

5   <u>For t=1</u>

Initially, a sample is randomly taken from each asset's expected annualised return distribution having a holding period of one year 62. Generation of random numbers is designed to allow for the well known cross correlation between asset returns. This results in a single expected return for each asset for the first year ($1ret^1_a$

10 and $1ret^1_b$).

A simulated single expected return for the investment portfolio at t=1 (22) is calculated by combining the assets in the weight proportions stipulated by the portfolio:

$$1ret^1_{ab} = (w_a * 1ret^1_a) + (w_b * 1ret^1_b)$$

A simulated expected wealth estimate at t=1 is obtained by combining the single

15 expected return with the initial capital (20):

$$wealth(1) = wealthcap(0) * (1 + 1ret^1_{ab})$$

The process of sampling from a one year holding period for each asset distribution and calculating a simulated expected wealth estimate at t=1 is repeated, say 1000 times 64, to produce a distribution of expected wealth for the portfolio after one

20 year.

From this distribution, the expected wealth and associated risk of the investment after one year may be assessed 66. The expected wealth is based on the central tendency of the distribution and the risk is based on the spread of the distribution. Risk can be expressed in different ways that are intuitive for investors. For example, for

25 various levels of confidence worst case wealth outcomes can be identified, as can best case and most likely expectations.

<u>For t=2</u>

To calculate the same parameters for the investment at t=2, the amount of time

30 separate portfolio components (i.e. the initial capital or the returns) will be invested must be determined. At t=2, there are two separate components of the investment differentiated by the length of time invested. The initial capital (wealthcap(0)) will be invested for two years (24) and the expected return on the portfolio generated from t =1 (r_{port}) (22) will be invested for one year (26). r_{port} is taken to be the equilibrium or long

35 run expected return of the portfolio calculated from the weighted sum of the component asset equilibrium returns.

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To calculate the return on the initial capital at t=2 (24), the expected annualised asset return distribution having a holding period of two years is sampled twice for the each asset 62:

$$2ret^1_a, 2ret^2_a$$
$$2ret^1_b, 2ret^2_b$$

Next, annualised expected wealth return on initial capital at t=2 for each asset is calculated according to the weight proportions stipulated by the portfolio 68:

$$wealthcap_a(2) = (w_a*wealthcap(0)) * (1 + 2ret^1_a) * (1 + 2ret^2_a)$$
$$wealthcap_b(2) = (w_b*wealthcap(0)) * (1 + 2ret^1_b) * (1 + 2ret^2_b)$$

An expected wealth from the initial capital that is invested for two years is then calculated by summing the weighted annualised expected wealth return for each asset:

$$wealthcap(2) = wealthcap_a(2) + wealthcap_b(2)$$

The expected return at t=2 from the return generated at t=1 (r_{port}) (26) is also calculated. Similarly as above, one sample is randomly taken from each asset's expected annualised return distribution having a holding period of one year ($1ret^1_a$, $1ret^1_b$). Then these samples are combined in the weight proportions stipulated by the portfolio:

$$1ret^1_{ab} = (w_a*1ret^1_a) + (w_b*1ret^1_b)$$

Then, the expected wealth on the return generated on the portfolio is calculated for t = 1 (r_{port}) at t = 2

$$wealthret(1) = (wealthcap(0)*r_{port}) * (1+1ret^1_{ab})$$

A single expected wealth estimate for t = 2 is then obtained by summing the returns on the two components, that is from the initial capital (wealthcap(2)) (24) and the expected return on the return generated at t=1 (26) (wealthret(1)):

$$wealth(2) = wealthcap(2) + wealthret(1)$$

The process of sampling the expected annualised return asset distributions to calculate the two returns on the initial capital (wealthcap(2)) and the expected return on the return generated at t=1 (wealthret(2)) is repeated, say 1000 times (64), to produce a distribution of expected wealth for the portfolio at t=2. From this distribution, the expected wealth and associated risk of the investment at t=2 may be assessed. Again, the expected wealth is based on the central tendency of the distribution and the risk is based on the spread of the distribution (66).

For t = 3

To calculate the same parameters for the investment at t=3, again the amount of time separate portfolio components (i.e. the initial capital or the returns) will be

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invested for must be determined. At t=3, there are three separate components of the investment differentiated by the length of time invested. The initial capital (wealthcap(0)) will be invested for three years (28), the expected return on the portfolio at t=1 will be invested for two years (30) and a return term at t=2 that includes the return on the return at t=1, will be invested for one year (32).

To calculate the return on the initial capital at t=3 (28), the expected annualised asset return distribution having a holding period of three years is sampled three times for the each asset 62:

$$3ret^1_a, \; 3ret^2_a, \; 3ret^3_a$$
$$3ret^1_b, \; 3ret^2_b, \; 3ret^3_b$$

Next, annualised expected wealth return on initial capital at t=3 for each asset is calculated according to the weight proportions stipulated by the portfolio:

$$wealthcap_a(3) = (w_a * wealthcap(0)) * (1+3ret^1_a) * (1+3ret^2_a) * (1+3ret^3_a)$$
$$wealthcap_b(3) = (w_b * wealthcap(0)) * (1+3ret^1_b) * (1+3ret^2_b) * (1+3ret^3_b)$$

An expected wealth from the initial capital is then calculated by summing the weighted annualised expected wealth return for each asset:

$$wealthcap(3) = wealthcap_a(3) + wealthcap_b(3)$$

The expected return at t=3 from the return generated at t=1 (30) is also calculated. Similarly as above, a sample is randomly taken from each asset's expected annualised return distribution having a holding period of two years 68:

$$2ret^1_a, \; 2ret^2_a$$
$$2ret^1_b, \; 2ret^2_b$$

Then these samples are combined in the weight proportions stipulated by the portfolio:

$$2ret^1_{ab} = (w_a * 2ret^1_a) + (w_b * 2ret^1_b)$$
$$2ret^2_{ab} = (w_a * 2ret^2_a) + (w_b * 2ret^2_b)$$

Then the return generated at t=1 from the initial capital is grown by these samplings:

$$wealthret(1) = (wealthcap(0) * r_{port}) * (1+2ret^1_{ab}) * (1+2ret^2_{ab})$$

The expected return on the return term of the portfolio at t=2 that is invested for one year (32) is also calculated 68. This term is $r_{port} + r_{port}^2$.

Firstly, a sample is taken from the distribution of each asset having a one year holding period ($1ret^1_a$, $1ret^1_b$). Then these samples are combined in the weight proportions stipulated by the portfolio:

$$1ret^1_{ab} = (w_a * 1ret^1_a) + (w_b * 1ret^1_b)$$

Then the expected return on the return term generated at t=2 is grown by these samplings 68:

$$wealthret(2)_1 = wealtcap(0) * (r_{port} + r_{port}^2) * (1+1ret^1_{ab})$$

A single expected wealth estimate for t=3 is then obtained by summing the three returns on the initial capital (wealthcap(3)), the expected return on the return generated at t=1 (wealthret(1)), and expected return on the returns on the portfolio from t=2 (wealthret(2)):

wealth(3) = wealthcap(3) + wealthret(1) + wealthret(2)

The process of sampling the expected annualised return asset distributions to calculate the returns from the initial capital and the expected return on the return generated at t=1 and t=2 is repeated, say 1000 times 64, to produce a distribution of expected wealth for the portfolio after at t=3. From this distribution, the expected wealth and associated risk of the investment at t=3 may be assessed. Again, the expected wealth is based on the central tendency of the distribution and the risk is based on the spread of the distribution 66.

This process can be adapted and repeated at the end of each period all the until the end of the investment term. The return-on-return terms become increasingly significant as the investment period is lengthened as they derive from the relationship $(1+r_{port})^n$. For example, at t=4 the return term generated at t=3 and having an investment horizon of one year would be $(r_{port} + 2*r_{port}^2 + r_{port}^3)$.

For simplicity, this example has not considered any additional capital contributions made during the investment term. To assess additional wealth and associated risk from financial contributions added to the portfolio at intervals through the investment life each contribution is considered as a separate component of the investment and is treated in the same way as the initial capital described above, starting from its time of introduction into the portfolio. Total portfolio wealth distributions are calculated at each interval by summing the wealth generated from initial capital and contributions.

Each calculated expected wealth outcome at the end of each period (including the individual samples) are written to file. The overall expected return and risk for the end of that investment term, say a term of three years, is determined by assessing all the stored simulation results having all the proportions of the asset allocation accounted for. For example, Fig. 7 shows an example of a resulting distribution of an investment in a superannuation fund derived from simulation when t=3 after 5000 simulation runs. The SAA of the investment being 40% Australian shares, 40% US shares, 10% direct property, the initial capital being $50,000 and the yearly contributions made from 9% of the investors salary of $50,000. This graph shows that the investor can expect a wealth within around $60,000 to $110,000 with the dispersion showing the risk (i.e. the relative frequency indicating the likelihood of each expected wealth).

Such a distribution can be produced to show the accumulated expected wealth and associated risk at any time within the investment term for any combination of assets (i.e. proportional combination of assets as stipulated in a strategic asset allocation) or financial products, and investor/member parameters. Such a distribution can be created for every period within the investment period (i.e. at the end of each year for a 30 year investment). The result of each of these combined creates a detailed wealth accumulation pathway for the investment over the entire investment term.

For example, Investor A is a member of a superannuation fund and has a long investment horizon. Other details include:

capital = $200

salary = $2000

contributions = 5% of salary

employer contributions = 9%

SAA = 60% Australian shares + 40% US Shares

Fig. 8 shows an example of an expected wealth growth over the first four years and upper and lower bounds which cover 70% of possible outcomes. This graph shows the expected wealth path that will fall somewhere between the upper and lower boundaries.

Clearly, the upper and lower bounds widen as time goes on. This is due to the increasing influence of investment risk over the predictability of capital and contributions (which dominate in the early years). If the investment is projected for the entire working life of the member, the bounds would be even further apart. This shows that it is important that members are informed of risk, and offered means to understand and to deal with it.

For some superannuation investors approaching retirement, it is important to avoid the widening of the bounds, that is an expanding risk envelope. For example, we now give Investor C a relatively large capital sum which it is important to protect a few years out from retirement. For example, Investor C may be a member of a superannuation fund and may be close to retirement. The Investor's details are:

capital = $5000

salary = $4000

contributions = 12% (combined employee and investor's contributions)

SAA = 20% shares plus 80% bonds

Investor C's wealth creation pathway is shown in Fig. 9. The boundaries are held in check towards the end of the holding period using this SAA.

The optimum wealth creation pathway can be selected by iteratively assessing multiple alternative SAAs using the above simulation method. The results of each simulated SAA can be compared and the SAA producing the optimum wealth creation pathway can be selected by considering the appropriate risk parameters.

5   For example, the wealth accumulation pathway of an SAA of a typical balanced fund or superannuation fund is compared with another asset allocation named Option A. This investor has initial capital of $50,000 and a salary of $50,000 from which regular contributions are made. Fig. 10 shows an example wealth distribution after one year for the Typical strategy and Fig. 11 shows a wealth distribution after one year for

10   the Option A strategy. The following table shows some risk measures derived from these distributions (i.e. the chance of meeting expected and lesser and better wealth outcomes):

Strategy	Lesser <$55,000	Expected >$60,000	Better >$65,000
Typical Strategy	3%	83%	45%
Option A Strategy	5%	78%	44%

15 It demonstrates that the distribution produced by the Typical Strategy in Fig. 10 is less risky than the distribution produced by the Option A Strategy on all three wealth-related risk measures.

The risk associated with the same investment strategies is again considered when the investment horizon is extended to five years. The resulting wealth

20   distributions for the Typical Strategy is shown in Fig. 12 and the Option A is shown in Fig. 13. The following table shows the same risk measures derived from these distributions having a five year investment horizon:

Strategy	Lesser <$90,000	Expected >$100,000	Better >$120,000
Typical Strategy	3%	74%	11%
Option A Strategy	1%	90%	13%

25 This shows that when the investment horizon is increased to five years, Option A is superior to the Typical strategy on all risk measures.

This can be repeated for multiple SAAs over various time horizons suitable to the investor allowing outcomes to be assessed over a large range of options.

The above example was for a member with substantial initial capital relative to

30 contributions. The following example shows outcomes for an investor with modest

initial capital of $5000 and salary of $50,000. Fig. 14 shows example wealth distributions after a five year investment horizon for the Typical Strategy and Fig. 15 shows the wealth distribution after a five year investment horizon for the Option A strategy. The following table shows some risk measures derived from these
5   distributions:

Strategy	Lesser <$36,000	Expected >$37,000	Better >$39,000
Typical Strategy	18%	61%	16%
Option A Strategy	11%	78%	18%

This investor is also favoured on all accounts by the strategy from allocation defined by Option A. Other presentations of risk include confidence levels for worst case end
10   wealth outcomes.

Rather than exhaustively try every possible SAA in order to discover the optimal SAA, optimisation can be modelled using the following method based on incremental diversification from an underlying asset.

The optimisation begins with selecting the underlying asset most appropriate to
15   the investor's circumstances (i.e. any expressed risk preferences or for a superannuation investment, the investor's time to retirement). Prudent limits are placed on exposure to any single asset.

For example, a long-horizon investor such as a twenty year old superannuation member in Australia would be assigned Australian Shares as the underlying asset –
20   providing maximum expected wealth for least risk amongst the list of assets normally considered over that horizon, without introducing any currency risk. Other equity assets are then added progressively (e.g. international shares), so as to further reduce risk while maintaining wealth to draw risk inside the tolerable level. As the rate of risk reduction begins to taper off and/or the prudential limits are reached the SAA is
25   obtained. For the case when equity assets are unable to draw risk down to the tolerance further diversification with other asset classes then proceeds at the margin.

Alternatively, for a superannuation investor who has only has a few years to retirement, the process is informed heavily by the risk appetite of the member (i.e. upper limit obtained through questionnaire, interview etc). For example, for a risk
30   averse member who has capital preservation as the priority the underlying asset could be Australian fixed interest with property and shares added sufficiently to maximise wealth within the upper limit on risk at retirement. But it could also be that some

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investors retain a long investment horizon on retirement so that long criteria continue to prevail even close to retirement.

Referring to Fig. 16, the present invention can also be used to allow a superannuation fund to focus on performance at a member level.

5   The board collects details on each of the fund's members (investors), especially the details that will effect the amount of risk that is suitable to the member when determining their SAA 50. Possible details include:

        age

        expected membership duration (i.e. retirement age, expected working life,

10 expected investment term)

        income

        current investment capital with the fund

        contribution amount

        prospective capital additions

15   wealth objectives

        any preferred risk tolerance for expected wealth

        other major investments

        taxation requirements

        and other special circumstances.

20   The information can be collected using methods such as forms or interviews. This information is then stored in a database.

        An optimal SAA for each investor is determined using the method described above 15.

        The expected investment life (i.e. retirement age) is a strong influential factor.

25 Taking investors A, B and C again, the following illustrative SAAs for each investor are devised.

	Time to Retirement (yrs)	Weight in Fund	Investor SAA
Investor A	30	20%	100% shares
Investor B	20	30%	100% shares
Investor C	3	50%	20% shares, 80% bonds

        The overall fund SAA is then determined using an aggregation of each investor's

30 SAA 54, which in this case would be:

        proportion of shares = 20% + 30% + (20% of 50%) = 60%

20

proportion of bonds = 0% + 0% + (80% of 50%) = 40%

Other considerations may also be made in determining a SAA for the fund, such as liquidity, asset limits and tax.

At t=2, the returns received from investments during t=1 are distributed to the investors according to each investors SAA 56.

If the whole fund experiences a return of say 9% net, the returns for each asset must also be determined. Assuming the returns were:

shares = 13%

bonds = 3.3%.

each investor's capital at t=2 will be the amount invested in each asset plus that amount's proportion of the asset return. For each investor:

investor A has $200 * 1.13 = $226 (from shares only)

investor B has $300 * 1.13 = $339

investor C has ($500 * 0.2) * 1.13 + ($500 * 0.8) * 1.033 = $526

The process of determining each investor's SAA and the fund's SAA is then repeated at t=2 as necessary adjustments may need to be made to fit the passage of time. For example, a change in any of the investor's details, a change in the investor's risk preferences, or a change in the predicted future return of an asset may alter any previously determined SAA 58. At 56, the return received during t=2 is then allocated accordingly at t=3.

Automatic and regular updating of each investor's SAA is also possible. This can be done automatically on the instructions of the investor to help insure that their investment is always optimal. This relieves unskilled investors of the concern that investment strategy might be inappropriate at any time.

Projections of risk to wealth creation over the expected investment life of the investor can be provided in advice to a member as described above 60. That way, the investor can get a better understanding of the through-life risk involved in the investment strategy. Given this projected pathway, the investor may require a new SAA be determined 52 (i.e. the investor may want to reduce or increase the risk shown in the projection).

It should also be noted from the example above that while the fund performing the prior art methods of investment allocations 'outperformed' the method of the embodiment of the present invention, (10% vs 9%), investors A and B gained more wealth from the preferred embodiment by taking more risk, which was entirely appropriate for them. Investor C's capital did not grow as much in the preferred embodiment but was not exposed to the risk of a significant loss of capital as in the

prior art method, at a point when capital losses would be difficult to retrieve. This further illustrates how vacuous the current practice of comparing fund returns is, without any insight into the member composition and needs.

The invention has advantages that should reorientate the management of
5   superannuation funds away from preoccupation with comparisons of short term performance which bring the interests of the superannuation fund or product provider to the fore, to concentrating solely on the interests of the investor. In this regard the first order interest of the investor is to maximise wealth and minimise the chance of lesser wealth outcomes. The invention enables a new approach to performance
10   reporting and accountability, against projected wealth accumulation pathways. The business schemes to derive from this invention include new approaches to funds management and its accountability, new advisory structures for superannuation funds and investors and the setting of standards for investment product providers relating to long term wealth creation and risk.

15   It will be appreciated by persons skilled in the art that numerous variations and/or modifications may be made to the invention as shown in the specific embodiments without departing from the spirit or scope of the invention as broadly described.

For example, investors are interested in the yield of a portfolio at certain times.
20   A variation of the invention is to supply estimated yield and associated risk at any time, deriving from the dividend yield of share markets, the coupon of bonds and rents of property assets in the portfolios. Other sampling techniques from series of asset returns, such as 'bootstrapping', could be another example of a variation.

The present embodiments are, therefore, to be considered in all respects as
25   illustrative and not restrictive. The spirit and scope of this invention is different from the art that prevails in the large industry that services investors seeking to accumulate wealth for investors.